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ABSTRACT

The first part of this paper discusses two principal forms elaborated upon in previous literature for describing the relation between per pupil operating costs and school enrollment size. The first of these forms indicates that average per pupil costs decline up to a point as enrollment increases, reach a minimum, and then rise with further school enrollment size increases. The other major school cost-size relationship form indicates that average costs do not reach a minimum,, but rather decline at a decreasing rate as enrollments increase. The following two sections elaborate on the interaction between these forms, school enrollment size, and cost distribtuion characteristics. The second section focuses on the relation between school size distribution and marginal and variable (or total) costs. Next, the author provides a formal proof of the dispersion-costs relationship presented in terms of a U-shaped marginal costs curve that initially declines and then rises. The paper concludes with a discussion of the analysis policy implications, which include the need to monitor demographic factors likely to influence school enrollment size distribution, and to plan school facilities to respond flexibly to changing enrollment patterns. The paper includes illustrative references to studies of Australian public school systems. (JBM)

# THE DISTRIBUTION OF SCHOOL SIZE: SOME COST IMPLICATIONS

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Australian Council for Educational Research

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## Introduction

A school system, whether it is organized on a district, state or national basis, comprises a collection of schools whose individual patterns of enrollment growth and decline are likely to be uneven. Accordingly, it is unlikely that the distribution by enrollment size of the schools contained within a school system will be stable over time. Since the total costs incurred by a school system will be largely determined by the costs of operating the individual schools in that system, and as there is considerable evidence that school enrollment size influences per pupil costs, changes in the distribution by enrollment size may be expected to have cost implications. This paper attempts to elaborate some of these implications.

A distribution of schools by enrollment size, like any distribution, may be characterized in terms of measures of central tendency, dispersion, skewness and kurtosis. Analysis of the cost implications of changes in the first two of these measures is the central concern of the paper. In order to discern the likely size and direction of the cost implications of changes in these parameters, it is necessary to be aware of the function which describes the relation between per pupil operating costs and school enrollment size in the school system under consideration. Accordingly, the first part of the paper is devoted to an examination of the two principal forms of this function which have been identified by the school cost-size literature. In the second and third sections the interaction between these forms, characteristics of the distribution of school enrollment size and costs is elaborated. The paper concludes with a brief discussion of some of the policy implications of the analysis. Throughout the paper, illustrative references are made to studies of the government (public) school systems of Australia.

## Studies of the School-Size Relationship

In principle, to isolate the impact of school enrollment size on per pupil school costs, it is necessary to determine the least-cost combination of inputs able to produce a prespecified level of educational output at each enrollment level (Cohn, 1975). If this procedure is followed, it is then possible, by controlling for differences in the level and quality of inputs and outputs, to estimate the relationship between school enrollment size and per pupil school costs. However, the conceptual and empirical difficulties associated with a procedure of the type just described have meant that few, if any, school cost-size studies have incorporated all of its elements. Such difficulties are not surprising since specification of a cost function of school enrollment size requires the prior specification of a production function of schooling, and the latter task has, thus far, proven largely resistant to research efforts.

As a result, most studies reported in the school cost-size literature do not incorporate cost functions in their true sense, but rather represent attempts to assess how various categories of educational expenditure vary with school enrollment size, sometimes with a narrow measure of student performance as a quality control, but more often with student numbers as the output proxy (Fox, 1981). Even within this less ambitious framework, difficulties of data collection have generally excluded expenditure on capital facilities from analysis. Accordingly, the cost curves so estimated provide little guidance for the longer term in which both capital and labour inputs are able to vary (ibid).

The conceptual and empirical difficulties of school cost-size research that have been outlined in general form above and which are discussed in more detail by Cohn (1975), Hind (1977) and Fox (1981) raise questions about its utility for policy purposes. However, confidence in the two major forms of school cost functions that have been identified is increased by the fact that their behaviour accords closely with the two major types of cost functions that have been identified for a wide range of public enterprises, private manufacturing and retail industries (Mansfield, 1975 reviews a number of such studies).

One major form of the relationship between per pupil school operating costs (hereinafter referred to as average costs or AC) and school enrollment that can be identified from the school-cost size literature is represented in Figure 1. Under this formulation, average costs are U-shaped which indicates that they decline as enrollments

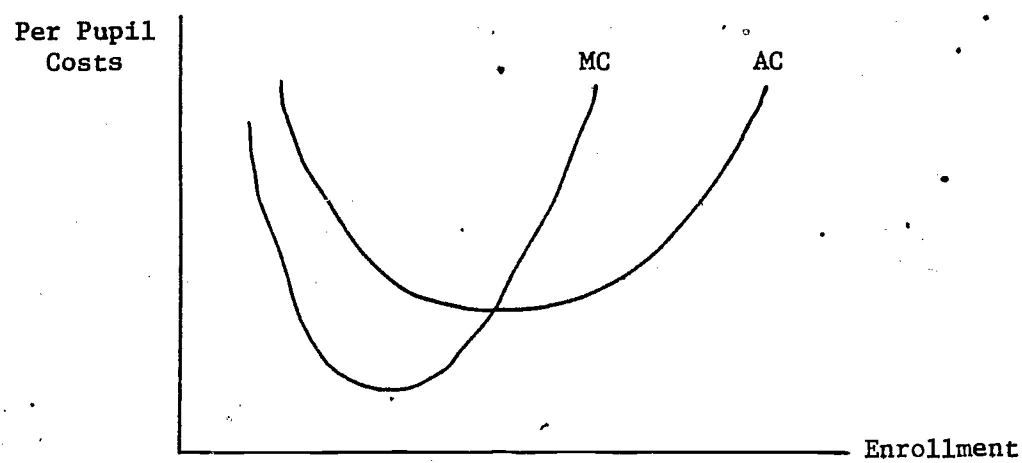


Figure 1. U-Shaped Average Cost Curve

increase up to a certain point, reach a minimum and then rise with further increases in school enrollment size. Average cost functions of this general form have been identified by Riew (1966) for Wisconsin high schools and by Cohn (1968) for Iowa high schools, amongst others (Fox, 1981 provides an extensive compilation of the relevant studies).

A U-shaped average cost curve will be described by the following functional form:

$$AC = a - bE + cE^2 \qquad (1)$$

where $E$ = school enrollment,

and $a$, $b$ and $c$ are constants, all of which $>0$.

Since average costs equal total costs divided by enrollments; equation (1) will have associated with it a total cost (TC) function as follows:

$$TC = aE - bE^2 + cE^3 \qquad (2)$$

Furthermore, since marginal costs (MC) are defined as the increase in total costs resulting from an increase in enrollment, a marginal cost function can be derived which is the first derivative of the total cost function:

$$MC = a - 2bE + 3cE^2 \qquad (3)$$

As shown in Figure 1, an AC curve which is U-shaped will have associated with it a marginal cost curve, also U-shaped, which initially lies below the AC curve, reaches a minimum and then rises to cut the AC curve at its minimum point. As will be elaborated in the next section, it is the properties of the MC curve which are of particular importance in assessing the cost implications of changes in the distribution of school enrollment size.

The other major form of the school cost-size relationship is shown in Figure 2 in which the AC curve is a rectangular hyperbola, indicating

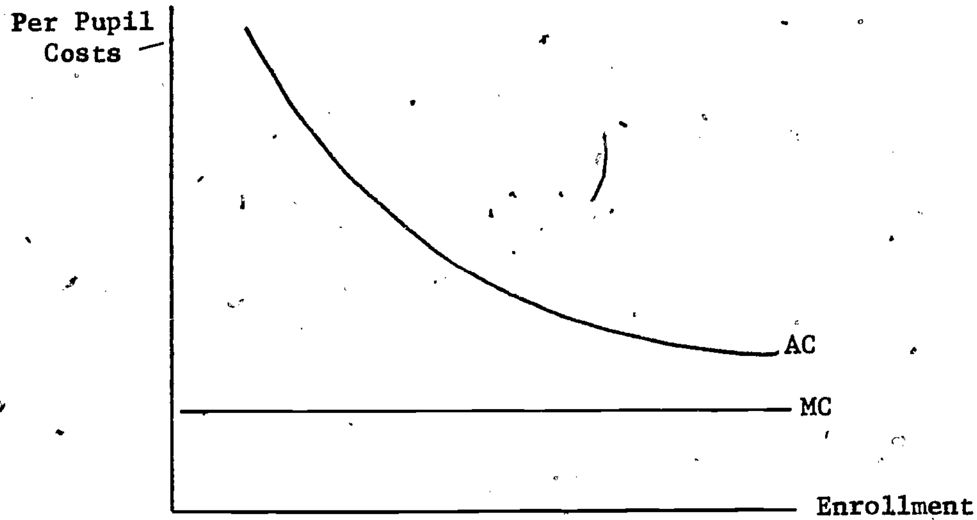


Figure 2. Hyperbolic Average Cost Function

that average costs do not reach a minimum, but rather decline at a decreasing rate as enrollments increase. Studies which have found such a curve to be the most powerful form of the school cost-size relationship include those of elementary and secondary schools in British Columbia (Wales, 1973) and rural primary (elementary) schools in New South Wales (Hind, 1977). The functional form which describes such a curve is as follows:

$$AC = d + fE^{-1} \qquad (4)$$

where d and f are constant and > 0.

The total cost function which applies to equation (4) has total costs as a linear function of enrollments:

$$TC = f + dE \qquad (5)$$

Accordingly, as marginal costs are the first derivative of total costs, the MC function in this instance is represented by a straight line:

$$MC = d \qquad (6)$$

As is shown in Figure 2, under such a formulation average costs approach, but never meet, the marginal cost line.

It is beyond the scope of this paper to evaluate the competing merits of two types of the AC curves described above as the most appropriate description of the behaviour of school costs. It may well be the case that a complete specification of the school cost-size relation in school systems which contain schools with very large enrollments could result in an AC curve which may be closer in shape to a flat-bottomed U-shaped curve with average costs relatively constant over a large part of the enrollment range. Such a curve would combine elements of both Figures 1 and 2.

Some support for this contention comes from Australia and New Zealand where the schedules by which government school systems allocate teachers to schools of different sizes result in marginal cost curves which decline as enrollments increase up to a certain point and then are relatively constant over the remainder of the enrollment range (McKenzie and Keeves, 1982). In addition, recent evidence indicates that school enrollment size may be negatively related to cognitive outcomes (Summers and Wolfe, 1977) and affective outcomes (Campbell, Cotterell, Robinson and Sadler, 1979). To the extent that these findings are valid and generalizable, they suggest that those studies which did not control for quality of output and which estimated an AC function of the type shown in Figure 2, if respecified could show that average costs eventually increase because additional resources are needed to maintain the quality of student outcomes. In the context of the present paper the importance of a flat-bottomed U-shaped AC curve is that it would be associated with an MC curve of a similar shape. That is, marginal costs could be expected to initially decline, remain relatively constant for a large part of the enrollment range and then to eventually rise above the AC curve.

The rest of the paper is predicated on the view that either the AC curve represented in Figure 1, or that shown in Figure 2 could best describe the behaviour of average costs in any particular school system. However, the cost implications of changes in the size distribution of schools will depend upon which type of function does apply in the school system under consideration since each implies a different form of MC function. It is to the relation between marginal costs and the distribution of school size that we now turn.

## Marginal Costs and the Distribution of School Size

The purpose of the preceding section was to indicate that studies of the school-cost size relation have produced estimates of AC functions which imply that the marginal cost curve may, on the one hand, be U-shaped, or on the other hand, may not change as school enrollment size increases. The shape of the marginal cost curve is critical in assessing the cost implications of changes in the distribution of school enrollment size since between any two enrollment levels the area beneath the MC curve measures the change in total costs (or variable costs) associated with a change in enrollments from one level to the other. This arises because the variable cost associated with a given change of output is the sum of the marginal cost of each incremental unit of output (Mahanty, 1980).

Figure 3 illustrates the principles involved. In the Figure are shown average and marginal cost curves of similar shape to those discussed in Figure 1. Area ABDE measures the increase in total costs associated

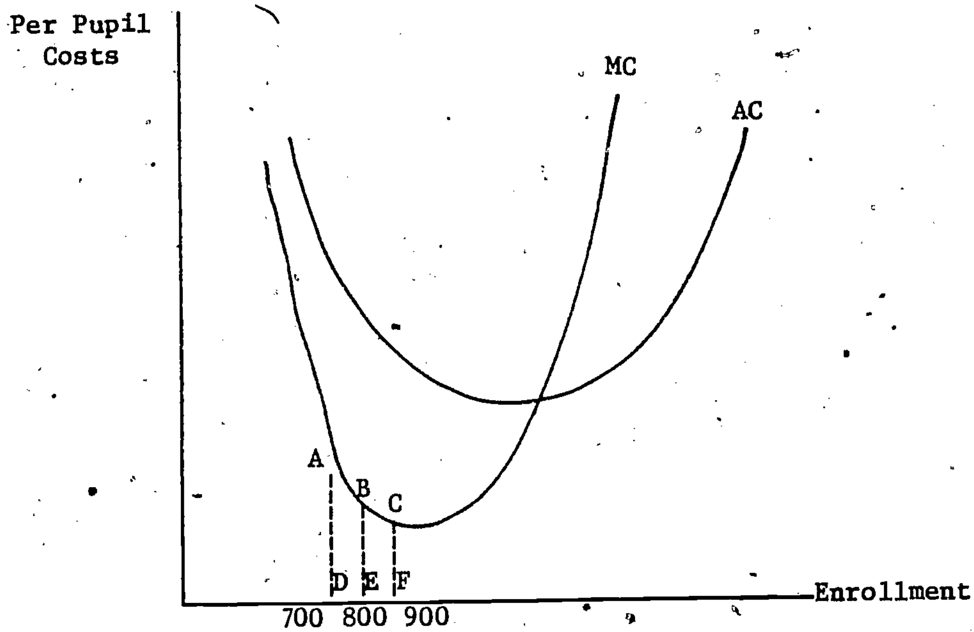


Figure 3. Marginal Cost Curve and Variable Costs

with an increase in school enrollments from 700 to 800 students. Similarly, area BCEF measures the decrease in total costs as school enrollments decline from 900 to 800 students. Since area ABDE exceeds area BCEF, this implies that given the cost relations pictured, it is less costly to conduct a school with 700 students and a school with 900 students than it is to operate two schools each containing 800 students. This position arises because the decrease in total cost associated with transferring 100 students from the school with 900 students (that is, area BCEF) is less than the consequent increase in total costs as the school with 700 students grows to 800 students (that is, area ABDE).

It was an illustration of this type that led Burke, Hudson and Gould (1981) to conclude that under conditions where the marginal cost of enrolling an additional student declines as school enrollment size increases, a reduction in the dispersion of the distribution of school size around the mean will increase per pupil school costs, other factors remaining equal. A formal proof of this relation is developed later in the paper.

The analysis of Burke et al (1981) only examined the influence of changes in the dispersion of school size on costs under the situation where marginal costs are declining. Figure 3 indicates, however, that where the AC curve is U-shaped, marginal costs eventually flatten out and then commence to rise as enrollments increase. Indeed, marginal costs may even be rising while average costs continue to fall.

It is a straightforward matter to extend the analysis to incorporate those situations in which marginal costs are either rising or constant. Over the enrollment range where marginal costs are rising, a reduction in the dispersion of school size could be expected to reduce costs since the decline in total cost associated with any given decrease in enrollment size will exceed the increase in total cost arising from an increase in enrollment size of the same magnitude. Over the enrollment range where marginal costs are constant, a change in the dispersion of school size should not affect total costs, other factors remaining constant, since the increase in total cost associated with any enrollment increase is equally matched by the decrease in total cost caused by an enrollment decrease of the same size. This implies that in those school systems where the school cost-size relationship could be characterized by the cost curves in Figure 2, changes in the dispersion of school size should not affect total costs, other factors remaining equal.

A Formal Proof

The proof of the relationship between dispersion and costs elaborated above is presented in terms of the U-shaped MC curve described by equation (3) since this function produces an MC curve which initially declines and then rises. To isolate the cost implications of changes in dispersion, it is assumed that average school size remains constant.

The area under the MC curve between any two enrollment levels represents the change in total costs associated with a change in enrollments between those two levels. Accordingly, the increase in total costs associated with a change in enrollments from e to e+1 is measured by

$$\int_{e}^{e+1} (MC)\, dE \qquad (7)$$

By substitution of equation (3), this expands to

$$\int_{e}^{e+1} (a - 2bE + 3cE^2)\, dE \qquad (8)$$

If there exist two schools, each with enrollment e, for the mean school size to be maintained an increase in enrollments at one school from e to e+1 must be matched by a decrease in enrollments at the other school from e to e-1. The decrease in total costs as enrollment declines from e to e-1 is given by

$$\int_{e-1}^{e} (a - 2bE + 3cE^2)dE \qquad (9)$$

Hence, the net change in total costs associated with the enrollment changes just described will be measured by

$$\int_{e-1}^{e} (a - 2bE + 3cE^2)\, de - \int_{e}^{e+1} (a - 2bE + 3cE^2) de \qquad (10)$$

Through expansion and cancellation, expression (10) reduces to 2b-6ce.If the simplified form of expression (10) is positive, this means that the decrease in total cost associated with the enrollment decline from e to e-1 exceeds the increase in total cost generated by a rise in enrollment size from e to e+1.

The next stage of the proof involves the calculation of a value for the simplified form of expression (10) under conditions when marginal costs are changing. The enrollment range over which marginal costs decline will be considered initially. Marginal costs decline in the enrollment range which lies to the left of the minimum point of the MC curve. Since, from equation (3)

$$MC = a - 2bE + 3cE^2 ,$$

$$\frac{dMC}{dE} = -2b + 6cE \qquad (11)$$

Marginal costs reach their minimum point when $\frac{dMC}{dE} = 0$, and which from expression (11) will be given by the point at which enrollments equal $\frac{b}{3c}$. Hence, the enrollment range under consideration in this example (namely, from e-1 to e+1) will be to the left of the minimum point of the MC curve if

$$e = \frac{b}{3c} - k - 1 \text{ where } k > o. \qquad (12)$$

When expression (12) is substituted in the simplified form of expression (10), the net result is $6c(k+1)$. Since both c and k are positive, this expression must be positive, which demonstrates that under conditions of declining marginal costs an increase in the dispersion of school size leads to a reduction in costs, other factors constant.

Under conditions where marginal costs are increasing, the range of enrollments under consideration lies to the right of the minimum point of the MC curve and therefore

$$e = \frac{b}{3c} + k + 1 \text{ where } k > o \qquad (13)$$

Substitution of (13) into expression (10) gives the net result $-6c(k+1)$ which must be negative. Thus, under conditions where the MC curve is rising, the magnitude of the decrease in total costs associated with a decline in enrollment is less than the size of the increase in total costs associated with an enrollment increase of equal magnitude. Therefore, where marginal costs are rising, an increase in the dispersion of school size increases costs, other factors constant.

## The Interaction of Average School Size and Dispersion

In the previous section the cost implications of changes in the dispersion of school size were investigated under conditions in which the average school size was constant. This procedure was adopted so that the independent effects of changes in dispersion could be addressed. In practice, however, it is more likely that average school size and dispersion will both be in a process of change over time. As a further complication, such changes need not necessarily be of the same order of magnitude, nor even in the same direction. The purpose of this section is to determine the likely cost implications of joint changes in average school size and the dispersion of school size around this average.

To commence, the cost implications of changes in average school size under conditions in which the dispersion of school size is held constant will be examined. To simplify the discussion, changes in the size of schools that lie in the enrollment range indicated by Figure 1 which shows a positive relation between enrollment size and per pupil costs will be excluded from analysis. The complexities introduced by a school system which comprises some schools that lie in the enrollment range where AC declines and others that lie where AC rises run the risk of obscuring the exposition of the general relations between mean size and dispersion which is the major purpose of this paper. However, as indicated by Figure 1, a U-shaped AC function implies that for at least part of the enrollment range where AC declines, marginal costs are rising and this position is not excluded from the following analysis. An increase in marginal costs is also characteristic of the enrollment range where the AC curve rises. Accordingly, the discussion of the impact of changes in the distribution of school size over the enrollment range where marginal costs rise, does provide some guidance for the likely cost implications of changes in the enrollment range where average costs rise. For the school systems which contain schools which lie along the full range of enrollments represented by a U-shape AC curve, a more complete analysis would require knowledge of this distribution so that the different cost implications of changes in school size could be appropriately weighted.

With the simplifying assumption that we are only concerned with that part of the enrollment range over which average costs decline as school enrollment size increases, the cost implications of changes in average school under conditions where the dispersion of school size remains constant can now be assessed. To assist this process, an example is provided which uses the AC function estimated by Cohn (1968) for Iowa high schools. The function estimated by Cohn was

$$AC = 390.05 - 0.1775\ E + 0.0000537\ E^2$$

This function can be used to compile a table of the costs of providing schools of different sizes:

| School Enrollment | Per Pupil Cost ($) | Total Cost ($) |
|---|---|---|
| 150 | 365 | 54750 |
| 300 | 342 | 102600 |
| 600 | 303 | 181800 |
| 750 | 287 | 215250 |
| 1200 | 254 | 304800 |

These cost data can be used to assess the impact on costs of changes in average school size under different assumptions about school size dispersion. This is done in the following table for a simple system comprising just two schools.

| Enrollment Size | Mean | Standard Deviation | Total Cost | Per Pupil Cost |
|---|---|---|---|---|
| 600, 1200 | 900 | 300 | 486600 | 270 |
| 150, 750 | 450 | 300 | 270000 | 300 |
| 300, 600 | 450 | 150 | 284400 | 316 |

As the table illustrates, although the 50 per cent decline is average school had, as expected, the effect of increasing per pupil costs, the increase was more marked when the proportionate decline in enrollment was spread equally amongst the two schools so that the absolute measure of dispersion declined. Since, as demonstrated in the previous section, over the enrollment range in which marginal costs decline, a reduction in dispersion increases costs, the net effect has been that the rise in average costs associated with a decline in average school size is exacerbated if the pattern of enrollment decline is spread amongst schools in such a way that dispersion is reduced. If, on the other hand, the enrollment decline occurred over an enrollment range in which marginal costs were increasing, the effect would be reversed: a reduction in dispersion would tend to offset some of the increase in per pupil costs.

It is possible to extend this type of analysis to all the possible combinations of changes in average school size and dispersion under conditions in which marginal costs are either falling or rising. In summary, it can be demonstrated that where marginal costs decline as enrollment size increases,

1. A reduction in dispersion will tend to exacerbate the increase in per pupil costs associated with a decline in average school size, and offset the decrease in per

pupil costs associated with an increase in average school size.

2. An increase in dispersion will tend to offset the increase in per pupil costs associated with a decline in average school size, and reinforce the decrease in per pupil costs associated with an increase in average school size.

Under conditions where marginal costs increase as enrollment size increases, the effects of changes in dispersion will work in the opposite direction to those just described.

There is some empirical support for these propositions provided by the behaviour of teacher salary costs in Australian government school systems. As documented by McKenzie and Keeves (1982), the schedules by which the number and seniority classifications of teachers in Australian government schools of different sizes are determined suggest marginal cost curves which exhibit a slight decline over a considerable part of the enrollment range. Accordingly, it could be expected that, for example, those Australian government school systems which experienced a reduction in both dispersion and average school size over the 1970's, per pupil costs could have been expected to rise considerably. Data presented by Burke et al (1981) suggests that this did occur. For example, between 1971 and 1980 the average primary school size in the Australian Capital Territory declined from 505 to 395 students and the student deviation of school size decreased from 196 to 158 over the same period. These changes were accompanied by an increase in per pupil costs greater than that which could have been expected on the basis of the decline in average school costs alone (ibid), which supports the contention that the reduction in dispersion exacerbated the increase.

## In Conclusion

The preceding analysis has highlighted the potential importance of changes in the dispersion of school size as a factor influencing changes in the per pupil costs of school systems. In practice, the extent to which changes in the dispersion of school size are likely to have important cost implications will depend initially upon the shape of the average cost function which applies to the system under consideration. For those school systems in which the allocation of teachers and other resources to school results in an average cost function of the type described in Figure 2, changes in the dispersion of school size are unlikely to have significant cost ramifications. However, it should be noted that the cost functions which have formed the basis for this analysis have been primarily based on studies of recurrent costs. To the extent that changes in the dispersion of school size severely overtax capital facilities at individual schools, the cost implications of course may be considerable, no matter what the shape of the curve relating recurrent costs and enrollments.

For those school systems characterized by cost functions of a form which indicates that there may be cost implications associated with changes in the distribution of school enrollment size, an awareness of such implications is likely to feed directly into educational policy formulation in several ways. First, it underlines the importance of an active monitoring of the demographic and other factors likely to influence the distribution of school enrollment size. Secondly, it promotes an awareness of the need to plan school facilities which are also to respond flexibly to changes in enrollment patterns. Thirdly, it concentrates attention on policies which may limit the potentially harmful consequences of increased educational costs generated by changes in the distribution of school size. Finally, an awareness of the cost implications of changes in the distribution of school enrollment size should lead to a more thorough search for the potential effects on the spread of school size of policies whose primary aim is not directly concerned with school size. For example, policies which increase parental freedom of choice in the selection of a school for their children could in some instances lead to an increase in school size dispersion as children transfer to more popular schools. On the other hand, if such policies are accompanies by greater autonomy for schools to develop specialized programs, average school size and dispersion may both be reduced. Assessment of the school size implications of such policies increases the likelihood that sufficient resources can be made available to ensure their success.

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